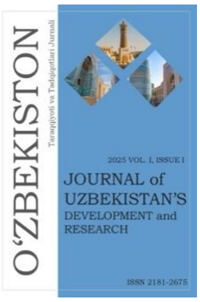

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YUKLANGAN HADLI KORTEVEG-DE FRIZ TENGLAMASINI DAVRIY FUNKSIYALAR SINFIDA INTEGRALLASH

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KEYWORDS

Davriy funksiyalar sinfi, davriy masala, teskari spektral masala, Shturm-Liuvill operatorlarining Lyapunov funksiyasi.

ABSTRACT

Hozirgi kunda nochiziqli to'lqinlar nazariyasidagi izlanishlar, jumladan, optik solitonlar telekomunikatsion texnologiyalar sohasida qo'llanila boshladi. Bundan tashqari, fizik sistemalarga ta'sir etuvchi kuchlar faqat vaqtning ma'lum bir davri mobaynida chegaralangan bo'ladi, shuning uchun haqiqiy modellar fazoviy o'zgaruvchilar bo'yicha davriy va deyarli davriy funksiyalar sinfidagi tenglamalarni o'rganishga keltiriladi. Shu munosabat bilan davriy funksiyalar sinfida yuklangan nochiziqli modifitsirlangan Korteveg-de Friz tenglamasini o'rganish maqsadli ilmiy tadqiqotlardan hisoblanadi. Bu paragrafda davriy koeffitsientli Shturm- Liuvill operatorlarining kvadratik dastasi uchun qo`yilgan teskari spektral masalaga oid kerakli ma'lumotlarni keltramiz .

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1-§. Lyapunov funksiyasining xossalari.

Butun o`qda berilgan ushbu

$$-y''+q(x)y=\lambda y,\ -\infty<x<\infty, \qquad (1.1)$$

Shturm-Liuvill masalasini ko`rib chiqamiz. Bu yerda $q(x)$ haqiqiy uzluksiz π -davrli funksiya. $c(x,\lambda)$ va $s(x,\lambda)$ orqali (1.1) tenglamaning ushbu

$$\begin{cases} c(0,\lambda)=1 \\ c'(0,\lambda)=0 \end{cases} \quad \text{va} \quad \begin{cases} s(0,\lambda)=0 \\ s'(0,\lambda)=1 \end{cases} \qquad (1.2)$$

boshlang`ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. Soddalik uchun quyidagi

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beliglashlarni kiritib olamiz:

$$c := c(\pi,\lambda),\ c' := c'(\pi,\lambda),\ s := s(\pi,\lambda),\ s' := s'(\pi,\lambda).$$

1-xossa. a) $c(x,\lambda)$ va $s(x,\lambda)$ funksiyalar x o`zgaruvchining har bir tayinlangan qiymatida λ parametrga nisbatan butun funksiyalardir.

b) $c(x,\lambda)s'(x,\lambda) - c'(x,\lambda)s(x,\lambda) = 1$ ayniyat bajariladi.

Teorema 1.1. Ushbu

$$\begin{cases} Ly \equiv -y'' + q(x)y = \lambda y \\ y(0) = y(\pi) \\ y'(0) = y'(\pi) \end{cases} \qquad (1.3)$$

masalaning xos qiymatlari haqiqiydir. Bu masalaga davriy masala deyiladi.

Teorema 1.2. Davriy masalaning xos qiymatlari $\Delta(\lambda) - 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi, bu yerda

$$\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda).$$

$\Delta(\lambda)$ funksiyaga (1.1) masalaning Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema 1.1′. Ushbu

$$\begin{cases} Ly \equiv -y'' + q(x)y = \lambda y \\ y(0) = -y(\pi) \\ y'(0) = -y'(\pi) \end{cases} \qquad (1.4)$$

masalaning xos qiymatlari haqiqiydir. Bu masalaga antidavriy masala deyiladi.

Teorema 1.2′. Antidavriy masalaning xos qiymatlari $\Delta(\lambda) + 2 = 0$ tenglamaning ildizlari bilan ustma-ust tushadi.

Natija. $\Delta(\lambda) - 2 = 0$ va $\Delta(\lambda) + 2 = 0$ tenglamalarning ildizlari haqiqiy.

Misol. (1.1) tenglamada $q(x) \equiv 0$ bo`lsin. U holda

$$c(x,\lambda) = \cos\sqrt{\lambda}x, \quad s(x,\lambda) = \frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}$$

bo`ladi. Bundan Lyapunov funksiyasi ushbu

$$\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda) = \cos\sqrt{\lambda}\pi + \cos\sqrt{\lambda}\pi = 2\cos\sqrt{\lambda}\pi$$

tenglik bilan berilishi kelib chiqadi.

$\Delta(\lambda) - 2 = 0$ tenglamaning ildizlarini, ya'ni davriy masalaning xos qiymatlarini topamiz:

$$2\cos\sqrt{\lambda}\pi - 2 = 0,$$

$$\cos\sqrt{\lambda}\pi = 1,$$

$$\pi\sqrt{\lambda} = 2\pi n, \quad n = 0, 1, 2, \dots,$$

$$\lambda = (2n)^2, \quad n = 0, 1, 2, \dots .$$

Demak, bu holda davriy masalaning $\lambda_0 = 0$ xos qiymati karrasiz, qolgan barcha xos qiymatlari ikki karrali ekan: $\lambda_{2n-1} = \lambda_{2n} = (2n)^2, \quad n = 1, 2, \dots$. Bu xos qiymatlarga quyidagi ortonormallangan xos funksiyalar mos keladi:

$$y_0(x) = \frac{1}{\sqrt{\pi}}, \quad y_{2n-1}(x) = \sqrt{\frac{2}{\pi}}\cos 2nx, \quad y_{2n}(x) = \sqrt{\frac{2}{\pi}}\sin 2nx, \quad n = 1, 2, \dots \ .$$

Endi esa, $\Delta(\lambda) + 2 = 0$ tenglamaning ildizlarini, ya'ni antidavriy masalani xos qiymtalarini topamiz:

$$2\cos\sqrt{\lambda}\pi + 2 = 0, \quad \cos\sqrt{\lambda}\pi = -1,$$

$$\sqrt{\lambda}\pi = \pi + 2\pi n,$$

$$\lambda = (2n+1)^2, \ n = 0, 1, 2, \dots \ .$$

Bu holda antidavriy masalaning barcha xos qiymatlari ikki karrali bo`ladi:

$$\mu_{2n} = \mu_{2n+1} = (2n+1)^2, \ n = 0, 1, 2, \dots \ .$$

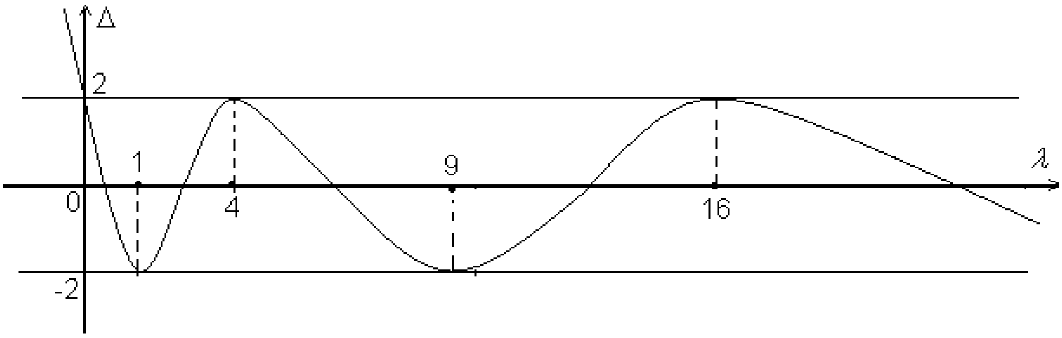
Lyapunov funksiyasining ushbu

$$\Delta(\lambda) = 2\cos\sqrt{\lambda}\pi, \ \lambda \ge 0,$$

$$\Delta(\lambda) = 2\,\mathrm{ch}\sqrt{|\lambda|}\pi, \ \lambda < 0$$

ifodalaridan uning grafigi quyidagicha bo`lishi ko`rinadi:

(1-rasm)



Endi $\Delta(\lambda)$ funksiyaning asimptotikasini o`rganamiz. Buning uchun avvalo, ushbu

$$\begin{cases} -c'' + q(x)c = \lambda c \\ c(0,\lambda) = 1 \\ c'(0,\lambda) = 0 \end{cases} \qquad (1.5)$$

$$\begin{cases} -s'' + q(x)s = \lambda s \\ s(0,\lambda) = 0 \\ s'(0,\lambda) = 1 \end{cases} \qquad (1.6)$$

masalalarga ekvivalent bo`lgan integral tenglamalar tuzib olamiz. Ular quyidagicha bo`ladi:

$$c(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}}q(t)c(t,\lambda)dt, \qquad (1.7)$$

$$s(x,\lambda)=\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\int_0^x \frac{\sin\sqrt{\lambda}(x-t)}{\sqrt{\lambda}}q(t)s(t,\lambda)dt. \qquad (1.8)$$

Bulardan hosila olsak,

$$c'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\int_0^x \cos\sqrt{\lambda}(x-t)q(t)c(t,\lambda)dt, \qquad (1.9)$$

$$s'(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x \cos\sqrt{\lambda}(x-t)q(t)s(t,\lambda)dt \qquad (1.10)$$

kelib chiqadi.

Lemma 1.1. $\sqrt{\lambda}=k=\sigma+i\tau$ bo`lsin, u holda quyidagi asimptotik formulalar o`rinlidir:

1) $c(x,\lambda)=\underline{\underline{O}}(e^{|\tau|x}),\ |k|\to\infty,\ x\in[0,\pi]$,

2) $c(x,\lambda)=\cos kx+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{|k|}\right),\ |k|\to\infty,\ x\in[0,\pi]$,

3) $c'(x,\lambda)=-k\sin kx+\underline{\underline{O}}(e^{|\tau|x}),\ |k|\to\infty,\ x\in[0,\pi]$,

4) $s(x,\lambda)=\underline{\underline{O}}\left(\frac{e^{|\tau|\chi}}{|\chi|}\right),|\chi|\to\infty,\ x\in[0,\pi]$,

5) $s(x,\lambda)=\frac{\sin kx}{k}+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{|k|^2}\right),\ |k|\to\infty,\ x\in[0,\pi]$,

6) $s'(x,\lambda)=\cos kx+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{|k|}\right),\ |k|\to\infty,\ x\in[0,\pi]$.

Natija 1. Ushbu

$$\Delta(\lambda)=2\cos\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{|\mathrm{Im}\sqrt{\lambda}|\pi}}{\sqrt{\lambda}}\right),|\lambda|\to\infty,$$

asimptotik tenglik o`rinli.

Natija 2. Ushbu

$$\Delta(\lambda)=2\cos\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right),\ \lambda\to+\infty$$

asimptotik formula o`rinli. Demak, $+\infty$ da $\Delta(\lambda)$ funksiyaning qiymatlari $[-2,2]$ kesma atrofida bo`ladi.

Natija 3. $\lambda<0$ bo`lsin, u holda $\sqrt{\lambda}=i\sqrt{|\lambda|}$ bo`ladi va

$$\Delta(\lambda) = 2ch\sqrt{|\lambda|}\pi + \underline{\underline{O}}\left(\frac{e^{\sqrt{|\lambda|}\pi}}{\sqrt{|\lambda|}}\right), \lambda \to -\infty$$

asimptotik formula bajariladi.

Agar $\lambda = (2m+1)^2$ desak, $\Delta(\lambda) - 2 = -4 + \underline{\underline{O}}\left(\frac{1}{m}\right)$ bo`ladi. Bu tenglikdan $\Delta(\lambda) - 2 = 0$ tenglama kamida bitta haqiqiy yechimga ega ekanligi ko`rinadi. Bu tenglamaning eng kichik ildizini λ_0 orqali belgilaymiz.

Teorema 1.3. Ushbu

$$\frac{d\Delta(\lambda)}{d\lambda} = \int_0^\pi \left\{c'(\pi,\lambda)s^2(t,\lambda) + [c(\pi,\lambda) - s'(\pi,\lambda)]c(t,\lambda)s(t,\lambda) - s(\pi,\lambda)c^2(t,\lambda)\right\}dt$$

formula o`rinli.

Teorema 1.4. A) λ son $\Delta(\lambda) = 2$ tenglamaning karrali ildizi bo`lishi uchun $s(\pi,\lambda) = 0,\ s'(\pi,\lambda) = 1,\ c(\pi,\lambda) = 1,\ c'(\pi,\lambda) = 0$ tengliklar bajarilishi zarur va yetarlidir.

B) λ son $\Delta(\lambda) = -2$ tenglamaning karrali ildizi bo`lishi uchun

$$s(\pi,\lambda) = 0,\ s'(\pi,\lambda) = -1,\ c(\pi,\lambda) = -1,\ c'(\pi,\lambda) = 0$$

tengliklar bajarilishi zarur va yetarlidir.

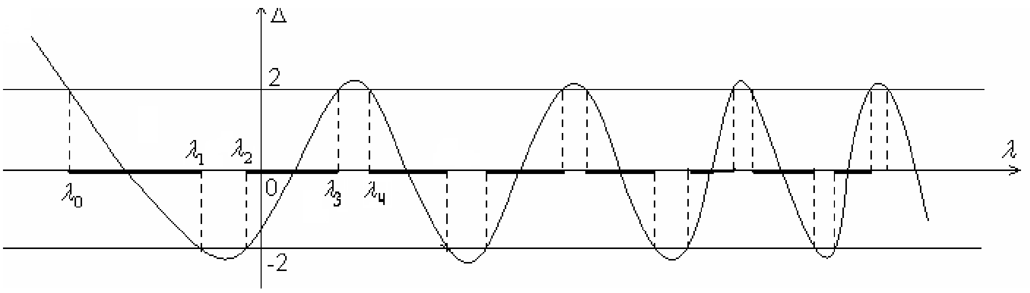
Teorema 1.5. A) Agar λ son $\Delta(\lambda) = 2$ tenglamaning karrali ildizi bo`lsa, $\ddot{\Delta}(\lambda) < 0$ bo`ladi. Xususan, bu nuqtada $\Delta(\lambda)$ funksiya lokal maksimumga erishadi.

B) Agar λ son $\Delta(\lambda) = -2$ tenglamaning ilidizi karrali bo`lsa, $\ddot{\Delta}(\lambda) > 0$ bo`ladi. Xususan, bu nuqtada $\Delta(\lambda)$ funksiya lokal minimumga erishadi.

Natija 1. $\Delta(\lambda) \pm 2 = 0$ tenglama ildizlarining karrasi ikkidan oshmaydi.

Natija 2. $\Delta(\lambda) - 2 = 0$ tenglamaning eng kichik ildizi karrasizdir. Haqiqatdan ham, agar u karrali bo`lsa, bu nuqtada lokal maksimum bo`lar edi, natijada bundan kichik ildiz topilgan bo`lar edi.

Natija 3. Yuqoridagi teoremalarni hisobga olib, Lyapunov funksiyasining grafigi quyidagicha bo`lishini ko`ramiz:



(2-rasm)

Ushbu

$$-y'' + q(x+t)y = \lambda y,\ x \in R \qquad (1.11)$$

tenglamani ko`rib chiqamiz, bu yerda $t$ haqiqiy parametr. (1.11) tenglamaning quyidagi $c(0,\lambda,t)=1$, $c'(0,\lambda,t)=0$, $s(0,\lambda,t)=0$, $s'(0,\lambda,t)=1$ boshlang`ich shartlarni qanoatlantiruvchi yechimlarini $c(x,\lambda,t)$ va $s(x,\lambda,t)$ orqali belgilaymiz. (1.11) tenglama uchun Lyapunov funksiyasi ushbu

$$\Delta(\lambda,t)=c(\pi,\lambda,t)+s'(\pi,\lambda,t)$$

formula bilan beriladi.

Teorema 1.6. Ushbu ayniyat o`rinli $\Delta(\lambda,t)\equiv\Delta(\lambda)$, ya'ni Lyapunov funksiyasi $t$ parametrga bog`liq bo`lmaydi.

Natija. Ma'lumki (1.11) masalaning spektri $-2\le\Delta(\lambda,t)\le 2$ tengsizlik bilan aniqlanadi. Teorema 1.6 ga ko`ra ushbu tengsizlikning yechimi $-2\le\Delta(\lambda)\le 2$ tengsizlikning yechimi bilan ustma-ust tushadi. Demak, (1.11) masalaning spektri $t$ parametrga bog`liq emas ekan, uning spektri (1.1) masalaning spektri bilan ustma-ust tushar ekan.

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